

Spin effects in $B_c \rightarrow X_{c\bar{c}}\pi(\rho)$ decays.

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Abstract

The two-particle hadronic decays of B_c meson into S-wave and P-wave charmonium states $X_{c\bar{c}}$ are considered in the framework of hard gluon exchange model. It is shown that decay width of B_c meson into S-wave charmonium states doubly exceeds one for P-wave states. In compare with the previous estimations we predict the enhancement by 8 % the branching ratio for the B_c decays into J/ψ in the two-particle hadronic decays via the contribution of the radiative decays of P-wave states in the cascade processes.

1 Introduction

The first observation of B_c meson at FNAL [1] motivates new theoretical study in the field of heavy quarkonium physics. The unique properties of B_c meson, containing heavy quarks of different flavours, may be studied, first of all, via decay modes with J/ψ meson in the final state. We consider here two-particle hadronic decays of B_c meson into S-wave and P-wave charmonium states

$$B_c \rightarrow X_{c\bar{c}} + \pi(\rho), \quad (1)$$

where $X_{c\bar{c}} = \eta_c$ or J/ψ for S-wave states, and $X_{c\bar{c}} = h_c, \chi_{c0}, \chi_{c1}, \chi_{c2}$ for P-wave states. These decays are of considerable interest as a clean signal of B_c meson production in high energy collisions [2].

Because of the large momentum transfer to the spectator quark ($|k^2| \sim m_c^2 \gg \Lambda_{QCD}$) in the decays (1.1), the hard scattering formalism is more appropriate than the spectator model, which based on the transition form-factor calculation under the overlapping of the quarkonium wave functions. In contrast to the spectator model, the hard scattering formalism results in the approximate double enhancement of the decay amplitude for the decays $B_c \rightarrow J/\psi(\eta_c)\pi$, as it was found in [3, 4].

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2 The model

In the model under consideration it is assumed that heavy quarkonium (B_c or $X_{c\bar{c}}$) is a non-relativistic quark-antiquark system with small binding energy. Such a way, the B_c meson decay amplitude is factorized in a hard part which describes the process $\bar{b} + c \rightarrow \bar{c} + c + \pi(\rho)$ and an amplitude describing the binding of the initial and the final heavy quark pairs into B_c meson and $X_{c\bar{c}}$ state.

The general covariant formalism for calculating the production and decay rates of S-wave and P-wave heavy quarkonium in the non-relativistic expansion was developed some times ago [5]. In the leading non-relativistic approximation, the mass of B_c meson m_1 is simply the sum of b-quark and c-quark masses $m_b + m_c$, and the mass of $X_{c\bar{c}}$ is equal $2m_c$. The amplitude for decay the bound-state ($\bar{b}c$) in a state with momentum p_1 , total angular momentum J_1 , total orbital momentum L_1 and total spin S_1 into bound-state ($\bar{c}c$) in a state with momentum p_2 , total angular momentum J_2 , total orbital momentum L_2 and total spin S_2 is given by

$$A(p_1, p_2) = \int \frac{d\vec{q}_1}{(2\pi)^3} \sum_{L_{1z} S_{1z}} \Psi_{L_{1z} S_{1z}}(\vec{q}_1) \langle L_1 L_{1z}; S_1 S_{1z} | J_1 J_{1z} \rangle \times \quad (2)$$

$$\times \int \frac{d\vec{q}_2}{(2\pi)^3} \sum_{L_{2z} S_{2z}} \Psi_{L_{2z} S_{2z}}(\vec{q}_2) \langle L_2 L_{2z}; S_2 S_{2z} | J_2 J_{2z} \rangle M(p_1, p_2, q_1, q_2),$$

where $M(p_1, p_2, q_1, q_2)$ is the hard amplitude which is described by the diagrams in Fig. 1,2.

To introduce the operators $\Gamma_{SS_z}(p, q)$ which projects the quark-antiquark pairs onto the bound states with fixed quantum number up to second order in q_1 and q_2 :

$$\Gamma_{S_1 S_{1z}}(p_1, q_1) = \frac{\sqrt{m_1}}{4m_c m_b} \left(\frac{m_c}{m_1} \hat{p}_1 - \hat{q}_1 + m_c \right) \hat{A}_1 \left(\frac{m_b}{m_1} \hat{p}_1 + \hat{q}_1 - m_b \right), \quad (3)$$

where $\hat{A}_1 = \gamma_5$ for $S_1 = 0$ and $\hat{A}_1 = \hat{\varepsilon}(S_{1z})$ for $S_1 = 1$;

$$\Gamma_{S_2 S_{2z}}^\dagger(p_2, q_2) = \frac{\sqrt{m_2}}{4m_c^2} \left(\frac{m_c}{m_2} \hat{p}_2 + \hat{q}_2 - m_c \right) \hat{A}_2 \left(\frac{m_c}{m_2} \hat{p}_2 - \hat{q}_2 + m_c \right), \quad (4)$$

where $\hat{A}_2 = \gamma_5$ for $S_2 = 0$, $\hat{A}_2 = \hat{\varepsilon}(S_{2z})$ for $S_2 = 1$ and $\varepsilon(S_{1z, 2z})$ are spin-one polarization four-vectors.

Using projection operators (2.2) and (2.3) the hard amplitude $M(p_1, p_2, q_1, q_2)$ may be presented as follows:

$$M(p_1, p_2, q_1, q_2) = \text{Tr} \left[\Gamma^\dagger(p_2, q_2) \gamma^\beta \Gamma(p_1, q_1) \mathcal{O}_\beta \right], \quad (5)$$

where for decay with π meson in final state

$$\mathcal{O}_\beta = \mathcal{O}_\beta^1 + \mathcal{O}_\beta^2, \quad (6)$$

$$\mathcal{O}_\beta^1 = \frac{G_F}{\sqrt{2}} \frac{16\pi\alpha_s}{3} V_{bc} f_\pi a_1 \hat{p}_3 (1 - \gamma_5) \left(\frac{-\hat{x}_1 + m_c}{x_1^2 - m_c^2} \right) \frac{\gamma_\beta}{k^2}, \quad (7)$$

$$\mathcal{O}_\beta^2 = \frac{G_F}{\sqrt{2}} \frac{16\pi\alpha_s}{3} V_{bc} f_\pi a_1 \frac{\gamma_\beta}{k^2} \left(\frac{-\hat{x}_2 + m_b}{x_2^2 - m_b^2} \right) \hat{p}_3 (1 - \gamma_5), \quad (8)$$

and

$$\begin{aligned} \hat{x}_1 &= \frac{m_b}{m_1} \hat{p}_1 + \hat{q}_1 - \hat{p}_3, & \hat{x}_2 &= \frac{m_c}{m_2} \hat{p}_2 + \hat{q}_2 + \hat{p}_3 \\ \hat{k} &= \frac{m_c}{m_2} \hat{p}_2 - \frac{m_c}{m_1} \hat{p}_1 - \hat{q}_1 + \hat{q}_2. \end{aligned}$$

The factor a_1 comes from hard gluon corrections to the four-fermion effective vertex. Since q_1/m_1 and q_2/m_2 are small quantities, we can expand $M(p_1, p_2, q_1, q_2)$ around $q_1 = q_2 = 0$ in a Taylor expansion:

$$\begin{aligned} M(p_1, p_2, q_1, q_2) &= M(p_1, p_2, 0, 0) + q_{1\alpha} \frac{\partial M}{\partial q_{1\alpha}} \Big|_{q_{1,2}=0} + q_{2\alpha} \frac{\partial M}{\partial q_{2\alpha}} \Big|_{q_{1,2}=0} + \\ &+ \frac{1}{2} q_{1\alpha} q_{1\beta} \frac{\partial^2 M}{\partial q_{1\alpha} \partial q_{1\beta}} \Big|_{q_{1,2}=0} + \dots \end{aligned} \quad (9)$$

Here the each term correspond to quantum numbers $L_1 = L_2 = 0$, $L_1 = 1$ and $L_2 = 0$, $L_1 = 0$ and $L_2 = 1$, and so forth. Thus for the S-wave ($L = 0$) and P-wave ($L = 1$) states, the amplitude $A(p_1, p_2)$ will depend on the quarkonium radial wave-functions through the following relations:

$$\int \frac{d\vec{q}}{(2\pi)^3} \Psi_{00}(\vec{q}) = \frac{R_s(0)}{\sqrt{4\pi}}, \quad (10)$$

$$\int \frac{d\vec{q}}{(2\pi)^3} \Psi_{1L_z}(\vec{q}) q_\alpha = -i \sqrt{\frac{3}{4\pi}} R'_p(0) \varepsilon_\alpha(p, L_z), \quad (11)$$

where $\varepsilon_\alpha(p, L_z)$ is the polarization vector for the spin-one particle.

In the case of production charmonium state 1P_1 one has

$$\sum_{L_{2z}} \varepsilon^\alpha(p_2, L_{2z}) < 1L_{2z}, 00 | 1, J_{2z} > = \varepsilon^\alpha(p_2, J_{2z}). \quad (12)$$

The summation over polarization may be done using following expression

$$\sum_{J_{2z}=-1}^1 \varepsilon^\alpha(p_2, J_{2z}) \varepsilon^\beta(p_2, J_{2z}) = \mathcal{P}^{\alpha\beta}(p_2), \quad (13)$$

where

$$\mathcal{P}^{\alpha\beta}(p_2) = -g^{\alpha\beta} + \frac{p_2^\alpha p_2^\beta}{m_2^2}.$$

In the case of production $^3P_J(J = 0, 1, 2)$ states one has

$$\begin{aligned} \sum_{S_{2z}, L_{2z}} \varepsilon^\alpha(p_2, L_{2z}) < 1L_{2z}, 1S_{2z} | J_2, J_{2z} > \varepsilon^\beta(S_{2z}) = \\ = \begin{cases} \frac{1}{\sqrt{3}}(g^{\alpha\beta} - \frac{p_2^\alpha p_2^\beta}{m_2^2}) \text{ for } J_2 = 0, \\ \frac{i}{\sqrt{2}m_2} \varepsilon^{\alpha\beta\mu\nu} p_{2\mu} \varepsilon_\nu(p_2, J_{2z}) \text{ for } J_2 = 1, \\ \varepsilon^{\beta\alpha}(p_2, J_{2z}) \text{ for } J_2 = 2. \end{cases} \end{aligned} \quad (14)$$

The polarization sums for $J_2 = 2$ is given by the following expression [6]:

$$\begin{aligned} \sum_{J_{2z}=-2}^2 \varepsilon_{\alpha\beta}(p_2, J_{2z}) \varepsilon_{\mu\nu}^*(p_2, J_{2z}) = \frac{1}{2}(\mathcal{P}_{\alpha\mu}(p_2) \mathcal{P}_{\beta\nu}(p_2) + \mathcal{P}_{\alpha\nu}(p_2) \mathcal{P}_{\beta\mu}(p_2)) - \\ - \frac{1}{3} \mathcal{P}_{\alpha\beta}(p_2) \mathcal{P}_{\mu\nu}(p_2) \end{aligned} \quad (15)$$

3 The results

Omitting the details of calculations we presented below the results for decay widths of ground-state ($\bar{b}c$) system into different states of charmonium plus π or ρ meson. In the limit of vanishing π meson mass we obtained the simple analytical formulae:

$$\Gamma(B_c \rightarrow \psi\pi) = \frac{128}{9\pi} F \frac{|R_2(0)|^2}{m_2^3} \frac{(1+x)^3}{(1-x)^5}, \quad (16)$$

$$\Gamma(B_c \rightarrow \eta_c\pi) = \frac{32}{9\pi} F \frac{|R_2(0)|^2}{m_2^3} \frac{(1+x)^3}{(1-x)^5} (x^2 - 2x + 3)^2, \quad (17)$$

$$\Gamma(B_c \rightarrow h_c\pi) = \frac{128}{3\pi} F \frac{|R_2'(0)|^2}{m_2^5} \frac{(1+x)^3}{(1-x)^7} (x^2 - x + 2)^2, \quad (18)$$

$$\Gamma(B_c \rightarrow \chi_{c0}\pi) = \frac{128}{9\pi} F \frac{|R_2'(0)|^2}{m_2^5} \frac{(1+x)^3}{(1-x)^7} (3x^3 - 12x^2 + 14x - 7)^2, \quad (19)$$

$$\Gamma(B_c \rightarrow \chi_{c1}\pi) = \frac{256}{3\pi} F \frac{|R_2'(0)|^2}{m_2^5} \frac{(1+x)^3}{(1-x)^5} (x^2 - x - 1)^2, \quad (20)$$

$$\Gamma(B_c \rightarrow \chi_{c2}\pi) = \frac{256}{9\pi} F \frac{|R_2'(0)|^2}{m_2^5} \frac{(1+x)^5}{(1-x)^7}, \quad (21)$$

where

$$x = \frac{m_2}{m_1}, \text{ and } F = \alpha_s^2 G_F^2 V_{bc}^2 f_\pi^2 |R_1(0)|^2 a_1^2.$$

To perform the numerical calculations we use following set of parameters: $G_F = 1.166 \times 10^{-5}$ GeV $^{-2}$, $\alpha_s = 0.33$, $V_{bc} = 0.04$, $f_\pi = 0.13$ GeV, $m_\pi = 0.14$ GeV, $m_{B_c} = 6.3$ GeV, $m_\psi = 3.1$ GeV, $m_{\eta_c} = 2.98$ GeV, $m_{h_c} = 3.5$ GeV, $m_{\chi_{c0}} = 3.4$ GeV, $m_{\chi_{c1}} = 3.5$ GeV, $m_{\chi_{c2}} = 3.55$ GeV, $|R_{s1}(0)|^2 = 1.27$ GeV 3 , $|R_{s2}(0)|^2 = 0.94$ GeV 3 , $|R'_{p2}(0)|^2 = 0.08$ GeV 5 .

With above mentioned set of parameters one gets the following result

$$\Gamma(B_c \rightarrow J/\psi + \pi) = 7.5 \times 10^{-15} a_1^2 \text{ GeV}. \quad (22)$$

The decay widths into different charmonium states plus π meson may be presented through decay width for $B_c \rightarrow J/\psi\pi$ as it is shown in Table 1.

$X_{c\bar{c}}$	$2S+1X_J$	$\frac{\Gamma(B_c \rightarrow X_{c\bar{c}}\pi)}{\Gamma(B_c \rightarrow J/\psi\pi)}$	$\frac{\Gamma(B_c \rightarrow X_{c\bar{c}}\rho)}{\Gamma(B_c \rightarrow X_{c\bar{c}}\pi)}$
J/ψ	3S_1	1.00	4.0
η_c	1S_0	1.17	3.2
h_c	1P_1	0.50	3.7
χ_{c0}	3P_0	0.29	3.6
χ_{c1}	3P_1	0.10	5.6
χ_{c2}	3P_2	0.28	4.3

Table 1.

The another source of J/ψ mesons is two-particle decay of B_c meson with ρ meson in the final state: $B_c \rightarrow X_{c\bar{c}}\rho$. The calculating for the decay widths $\Gamma(B_c \rightarrow X_{c\bar{c}}\rho)$ may be done the same way as for widths $\Gamma(B_c \rightarrow X_{c\bar{c}}\pi)$ using substitution $f_\pi \hat{p}_3 \rightarrow m_\rho f_\rho \hat{\varepsilon}_3$ in (2.6) and (2.7), where ε_3^μ is ρ meson polarization four-vector. Taking into account that $f_\rho = 0.22$ GeV and $m_\rho = 0.77$ GeV, we have obtained the decay widths $\Gamma(B_c \rightarrow X_{c\bar{c}}\rho)$ which are presented in the Table 1 too as the ratio $\Gamma(B_c \rightarrow X_{c\bar{c}}\rho)/\Gamma(B_c \rightarrow X_{c\bar{c}}\pi)$.

We found surprisingly large value for the decay width of B_c meson into P-wave charmonia, which is 50 % of the decay width into S-wave states. Because of J/ψ meson production in the decays of B_c meson is very suitable process from viewpoint of an experimental study [1],

it is interesting to compare the direct J/ψ production ($B_c \rightarrow J/\psi\pi(\rho)$) and the cascade J/ψ production rates. The second one comes from radiative decays of the P-wave states χ_{c0} , χ_{c1} and χ_{c2} , which have following branching ratios into J/ψ plus γ : $\text{Br}(\chi_{c0} \rightarrow J/\psi + \gamma) = 0.007$, $\text{Br}(\chi_{c1} \rightarrow J/\psi + \gamma) = 0.27$ and $\text{Br}(\chi_{c2} \rightarrow J/\psi + \gamma) = 0.14$ [7]. Thus we have obtained

$$\frac{\Gamma(B_c \rightarrow \chi_{c0,c1,c2}\pi \rightarrow J/\psi\gamma)}{\Gamma(B_c \rightarrow J/\psi\pi)} = 0.068 \quad (23)$$

and

$$\frac{\Gamma(B_c \rightarrow \chi_{c0,c1,c2}\rho \rightarrow J/\psi\gamma)}{\Gamma(B_c \rightarrow J/\psi\rho)} = 0.082 \quad (24)$$

The ratio for the sum of the B_c meson decay widths into J/ψ meson plus π or ρ meson is equal to

$$\frac{\Gamma(B_c \rightarrow J/\psi\pi(\rho), \text{cascade})}{\Gamma(B_c \rightarrow J/\psi\pi(\rho), \text{direct})} = 0.08 \quad (25)$$

In compare with the previous estimations [3, 4] we should expect the enhancement by 8 % the branching ratio for the B_c decays into J/ψ in the two-particle hadronic decays via the cascade processes. This fact makes the probability of B_c meson observation through the decays $B_c \rightarrow X_{c\bar{c}}\pi(\rho)$

in the current experiments more real.

The author thanks V.V. Kiselev and A.K. Likhoded for the valuable discussion. This work is supported by the Program "Universities of Russia", Grant 02.01.03 and the Russian Ministry of Education, Grant 98-0-6.2-53.

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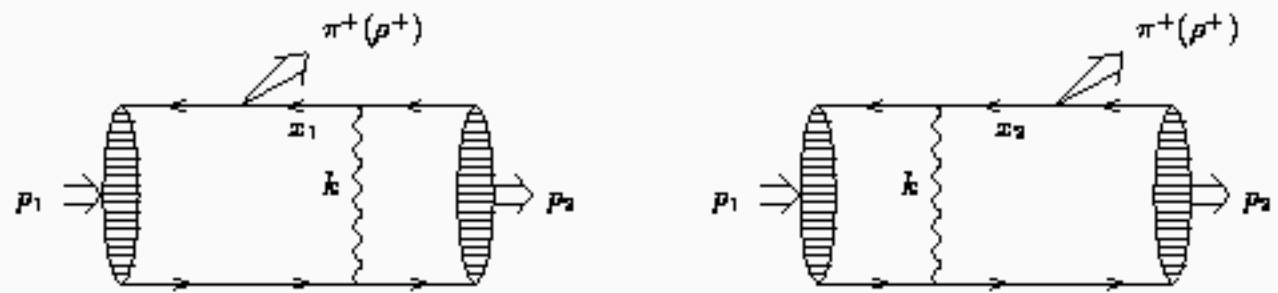


Fig.1 Diagrams for the decay $B_c \rightarrow X \pi$.